RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, JUNE 2022

SECOND YEAR (BATCH 2020-23)

Date : 25/06/2022 **MATHEMATICS (Honours)**

Time : 11.00 am – 1.00 pm **Paper : X [CC10]** Full Marks : 50

Group – A

Answer **any two** questions. [2×10]

1. a) Define astatic equilibrium. Find the necessary and sufficient condition for an equilibrium to be astatic. [1+5]

   b) A heavy particle rests on a rough parabola with its axis vertical and vertex downwards. If the latus rectum of the parabola is 4a, find the greatest altitude above the vertex at which the particle can remain at rest, the coefficient of friction being μ. [4]

2. a) A heavy elastic string whose natural length is $2\pi a$ is placed round a smooth cone whose axis is vertical and whose semi-vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is $a\left(1+\frac{W}{2\pi\lambda}.\cot\alpha\right)$. [5]

   b) A solid hemi-sphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ,ϕ be the inclinations of the string and the plane base of the hemi-sphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. [5]

3. a) Prove that in a conservative field, the sum of the kinetic and potential energies of a system is constant. [6]

   b) A sphere of weight W and radius r lies within a fixed spherical shell of radius R and a particle of weight w is attached to its highest point. Show that the equilibrium will be stable if $W \geq \frac{R-2r}{r} w$. [4]

Group – B

Answer **all** questions, maximum you score 30 marks.

4. Find the tangential and normal components of velocity and acceleration of a particle which describes a plane curve. [6]

5. A particle describes a plane curve under the action of a central force P per unit mass. Prove that, in usual notation, the differential equation of the path of the particle is $\frac{h^2}{p^3}\frac{dp}{dr} = P$. [6]

6. A particle rests in equilibrium under the attraction of two centres of force which attract directly as the distance, their intensities being μ and μ′. The particle is slightly displaced towards one of them, show that the time of small oscillation is $\frac{2\pi}{\sqrt{\mu+\mu'}}$. [4]

7. A particle is projected from the earth's surface vertically upwards with a velocity V. If h and H be the greatest heights attained by the particle moving under uniform and variable accelerations respectively, then show that $\frac{1}{h}-\frac{1}{H}=\frac{1}{R}$, where R is the radius of the earth. [5]

8. Find the law of force to the pole when the path is the cardioide $r=a(1-\cos\theta)$ and prove that if F be the force at the apse and ν be the velocity, then $3v^2=4aF$. [5]

9. If V_1 and V_2 be the respective velocities of a planet when it is nearest and farthest from the Sun, then prove that $(1-e)V_1=(1+e)V_2$, where e is the eccentricity of the planet's orbit. [4]

10. A particle of unit mass is projected with a velocity V at an angle α above the horizon in a medium whose resistance is k times the velocity of the particle. Show that the direction of its velocity will make an angle $\frac{\alpha}{2}$ above the horizon after a time

$$\frac{1}{k}\log\left(1+\frac{kV}{g}\tan\frac{\alpha}{2}\right).$$ [6]

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